TIME REVERSAL OF THE INCREASING GEOMETRICAL PROGRESSION OF THE POPULATION OF A SIMPLE BIOLOGICAL SPECIES

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Abstract

In this work we consider time reversal of the increasing geometrical progression of the population of a simple biological species without any enemies (predators) in the appropriate environment with unlimited resources (food, territory, etc.). It is shown that such time reversal corresponds to appearance of the cannibalism, i.e. self-predaciousness or self-damping phenomena which can be described by a type of difference Verhulst equation.

In this work we shall consider time reversal of the increasing geometrical progression of the population of a simple biological species without any enemies (predators) in the appropriate environment with unlimited resources (food, territory, etc.). It will be shown that such time reversal corresponds to appearance of the cannibalism, i.e. self-predaciousness or self-damping phenomena which can be described by a type of difference Verhulst equation. Also, it will be shown how such time reversal of the population dynamics can be demonstrated formally in an environment with changeable food concentration.

Suppose that an individual of a simple, monosexual biological species (eg. a bacterium species), in an appropriate environment with unlimited resources (food, territory, etc.) and without any enemies (predators), during a reproduction time interval (cycle), Δt, does a splitting (fision) in $k+1$ new individuals, where k represents a natural number 1,2,3... Presented dynamical change of the population (number of the individuals of given species) by interaction with environment during one reproduction time interval can be described by the following difference equation

$$\frac{\Delta p}{\Delta t} = ap_{in} \tag{1}$$

Here a is positive and it represents a constant that describes phenomenologically positive interaction between individual and environment. (By positive interaction between individual and environment there is increase of the population.) It can be presented in the following form

$$a = \frac{k}{\Delta t} \tag{2}$$

Also,

$$\Delta p = p_{fin} - p_{in} \tag{3}$$

where p_{in} and p_{fin} represent initial (at the beginning of the reproduction cycle) and final (at the end of the reproduction cycle) population in the general solution of (1). As it is not hard to see given general solution of (1), i.e. of

$$\frac{\Delta p}{\Delta t} = \frac{k}{\Delta t} p_{in} \tag{4}$$

is

$$p_{fin} = (k+1)p_{in} \tag{5}$$

which represents a geometrical progression with coefficient $1+k$ greater than 1. Supposed especial solution of (1), i.e. (4) that satisfies initial condition

$$p_{in} = 1 \tag{6}$$

is

$$p_{fin} = k+1 \tag{7}$$

Population dynamical equation (1), i.e. (4) holds a very important characteristics that can be called *locality*. Namely, population change during reproduction time interval (left hand-side of (1), i.e. (4)) represents a function (right hand-side of (1), i.e. (4)) that depends of the initial population and that does not depend of the final population. Or, given population dynamics does not need that final population is known a priori, but this final population can be determined by initial population and population dynamical equation (1), i.e. (4). Simply speaking, there is a finite speed of the population dynamical evolution form the past toward future. In case that population change during reproduction time interval in a population dynamical equation represents a function that depends of the final population too it can be said that corresponding population dynamics is non-local or instantaneous. Such population dynamics needs, in fact, that both initial and final population are known simultaneously. But such non-local population dynamics does not exist really within ecology.

It is not hard to see that locality of the population dynamics corresponds conceptually to locality of the physical dynamics. Simply speaking locality of the physical dynamics means in fact that there is no physical interaction faster than speed of light.

Now, apply time reversal transformation, T, [1], [2] that changes discretely time moment t in $t' = -t$, or initial in the final conditions in the general solution of an equation, at (1), i.e. (4). It yields

$$\frac{p'_{in} - p'_{fin}}{\Delta t} = \frac{k}{\Delta t} p'_{fin} \tag{8}$$

where p'in and p'fin represent initial (at the beginning of the reproduction cycle) and final (at the end of the reproduction cycle) population in the general solution of (8). As it is not hard to see given general solution is

$$p'_{fin} = \frac{1}{k+1} p'_{in} \tag{9}$$

which represents a geometrical progression with coefficient 1/(1+k) smaller than 1. For especially chosen

$$p'_{in} = k + 1 \tag{10}$$

it follows, as an especial solution of (8),

$$p'_{fin} = 1 \tag{11}$$

It is not hard to see that geometrical progression (9) represents generally the inversion of the geometrical progression (5), so that, in the especial case, it follows that (6) is numerically equivalent to (11) and (7) to (10). But, equation (8) can be transformed in the equivalent equation

$$\frac{\Delta p'}{\Delta t} = -\frac{k}{\Delta t} p'_{fin} = -a p'_{fin} \tag{12}$$

Here -a is negative and it represents a constant that describes phenomenologically negative interaction between individual and environment. (By negative interaction between individual and environment there is decrease of the population.) Also, here

$$\Delta p' = p'_{fin} - p'_{in} \tag{13}$$

But, in distinction from (1), i.e. (4), equation (12) is non-local. For this reason equation (12) representing time reversal of (1), i.e. (4) has not real ecological sense.

Suppose however that splitting of the individual during one reproduction cycle is recorded by a video camera and that later a play back backward is done. It formally corresponds to application of T at (1), i.e. (4). Then one can observe how at the "beginning" there are $k + 1$ individuals which interact mutually during time interval Δt so that only one individual stands while k other individuals disappear at the "end" of this interval. Simply speaking, effectively, one of $k + 1$ initial individuals kills (eats) all k other individuals. In this way here an effective cannibalism, i.e. self-predaciousness or self-damping phenomena within given species appears. If given species is sufficiently simple then cannibalism within given species can be biologically admirable.

Corresponding population dynamics that includes cannibalism , i.e. self-predaciousness or self-damping appearance, can be presented by the following local difference equation

$$\frac{\delta p'}{\Delta t} = \frac{k}{\Delta t} p'_{in} - \frac{b}{\Delta t} (p'_{in})^2 \tag{14}$$

where b represents self-damping constant. Obviously, (14) represents a type of the difference Verhulst equation [3]-[8]. Self-damping constant b can be determined from (14), (13) and conditions (10), (11). It yields

$$b = \frac{k(k+2)}{(k+1)^2} \tag{15}$$

Thus, both equations (1), i.e. (4) and (14) have, at least effectively, real ecological sense. Also (14) can be consistently effectively treated as time reversal of (1), i.e. (4), and vice versa. For this reason it can be consistently stated that set of given two equations (1), i.e. (4) and (14) is T invariant (symmetric) or time reversible (even if neither (1), i.e. (4) nor (14) is time reversible).

But, as it is well-known, time reversal does not correspond to any real ecological interaction. Nevertheless, instead of time reversal an analogous discrete transformation of (1), i.e. (4) with equivalent consequences can be done.

Namely, suppose that b represents a discrete function of the food concentration, C, etc. in the environment so that

$$b = 0 \qquad \text{for} \qquad C \geq C_0 \tag{16}$$

$$b = \frac{k(k+2)}{(k+1)^2} \qquad \text{for} \qquad C < C_0 \tag{17}$$

where C_0 represents the critical value of C. In this case solution of (14) under condition (16) and initial condition

$$p'_{in} = 1 \tag{18}$$

equivalent to (6) is

$$p'_{fin} = k + 1 \tag{19}$$

equivalent to (7), while, numerically inversely, solution of (12) under condition (17) and initial condition (10) is equivalent to (11).

It can be very interesting that theoretical results (14)-(17) be compared with empirical data, i.e. with results of the experimental analysis of the critical value of the food concentration in environment for simple biological species (eg. bacterium species etc.) that admits more or less discrete appearance of the cannibalism for food concentration smaller than critical. Especially it can be interesting for

$$k = 1 \tag{20}$$

when, according to (19),

$$b = 3/4 = 0.75 \tag{21}$$

In conclusion the following can be repeated and pointed out. Population dynamics of a simple biological species, living without any enemies, in an appropriate environment with unlimited resources, corresponding to increasing geometrical progression, turns by time reversal in a population dynamics corresponding to difference Verhulst population dynamics. All this can be demonstrated formally in an environment with changeable food concentration.

1 References

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